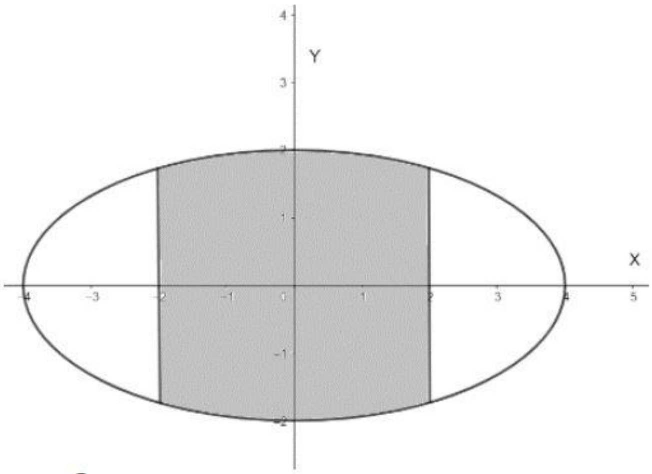
Applications Of The Integrals

1. Using integration, find the area of the ellipse $x^2/16 + y^2/4 = 1$, included between the lines x = – 2 and x = 2. (2024)

Ans.



$$\text{Area} = 4\int_0^2 y\,dx$$

$$= 4\left[\frac{1}{2}\int_0^2 \sqrt{4^2 - x^2}\,dx\right]$$

$$= 2\left[\frac{x}{2}\sqrt{4^2 - x^2} + 8\ \sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$$

$$= 2\left[\sqrt{12} + \frac{8\pi}{6}\right] = 4\sqrt{3} + \frac{8\pi}{3}$$

Previous Years' CBSE Board Questions

8.2 Area under Simple Curves

SA I (2 marks)

1. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration. *(2023)*

2. Using integration, find the area bounded by the curve $y^2 = 4x$, y-axis and $y = 3$. *(2021C)* Ev

3. Using integration, find the area of the region bounded by the line $2y = -x + 8$, x-axis, $x = 2$ and $x = 4$. *(2021C)* Ev

SA II (3 marks)

4. Find the area of the following region using integration. $\{(x, y) : y^2 \le 2x \text{ and } y \ge x - 4\}$ *(2023)*

5. Using integration, find the area of the region bounded by $y = mx$ $(m > 0)$, $x = 1$, $x = 2$ and the x-axis. *(2023)*

6. Using integration, find the area of the region $\{(x, y) : y^2 \le x \le y\}$. *(Term II, 2021-22)* Ev

LA I (4 marks)

7. Using integration, find the area of the region $\{(x, y) : 4x^2 + 9y^2 \le 36, 2x + 3y \ge 6\}$. *(Term II, 2021-22)* Ev

8. Using integration, find the area of the region bounded by lines $x - y + 1 = 0$, $x = -2$, $x = 3$ and x-axis. *(Term II, 2021-22)* Ev

9. If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration find the value of a, where $a > 0$. *(Term II, 2021-22)* Ev

10. Find the area of the region bounded by curve $4x^2 = y$ and the line $y = 8x + 12$, using integration. *(Term II, 2021-22)* Ev

11. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration. *(Term II, 2021-22)* Cr

LA II (5/6 marks)

12. Using integration, find the area of the region bounded by the circle $x^2 + y^2 = 16$, line $y = x$ and y-axis, but lying in the 1st quadrant. *(2023)*

13. Find the area of the following region using integration: $\{(x, y) : y < |x| + 2, y > x^2\}$ *(2020)* Ev

14. Using integration, find the area of a triangle whose vertices are (1, 0), (2, 2) and (3, 1). *(2020)* Ev

15. Using integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$. *(2020)* Ev

16. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. *(2020, NCERT, 2018, Delhi 2014)* Ev

17. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a using integration. *(2020C)* Ev

18. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$. *(2019)* Ev

19. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration. *(Delhi 2017)* Cr

20. Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$. *(AI 2017, 2015C)*

21. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y – axis. *(Foreign 2015)*

22. Find the area of the region in the first quadrant enclosed by the y-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$, using integration. *(NCERT, Delhi 2015C)* Cr

23. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$. *(Foreign 2014)* Cr

24. Using integration, find the area of the region bounded by the curves: $y = |x + 1| + 1$, $x = -3$, $x = 3$, $y = 0$ *(Delhi 2014C)*

25. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. *(Delhi 2014C)*

26. Using integration, find the area of the region in the first quadrant enclosed by the x – axis, the line $y = x$ and the circle $x^2 + y^2 = 18$. *(AI 2014C)*

CBSE Sample Questions

8.2 Area under Simple Curves

VSA (1 mark)

1. Find the area bounded by $y = x^2$, the x- axis and the lines $x = -1$ and $x = 1$. *(2020-21)* Ev

SA I (2 marks)

2. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$. *(2020-21)* Ev

SA II (3 marks)

3. Find the area of the region bounded by the curves $x^2 + y^2 = 4$, $y = \sqrt{3}x$ and x-axis in the first quadrant. *(2020-21)*

4. Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration. *(2020-21)* Cr

LA I (4/5 marks)

5. Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2, 0 \le y \le x, 0 \le x \le 2\}$ and find the area of the region using integration. *(2022-23)* Ev

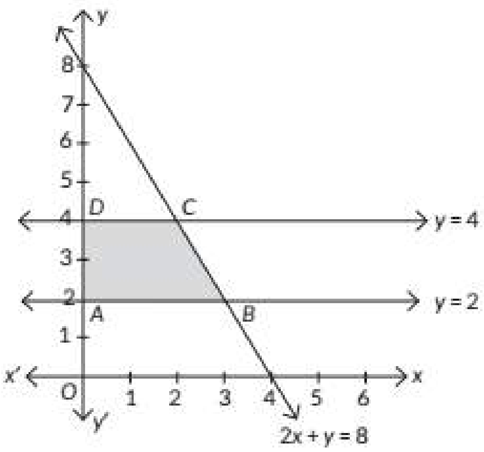
6. Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$, the parabola $y^2 = x$ and the x-axis. *(Term II, 2021-22)* Ev

7. Using integration, find the area of the region $\{(x,y): 0 \le y \le \sqrt{3}x, x^2 + y^2 \le 4\}$. *(Term II, 2021-22)* Cr

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. From the graph, $ABCD$ is the required region.



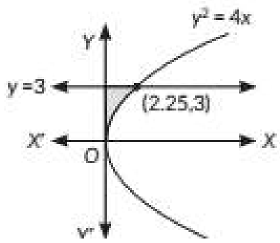
Now, area $= \int_2^4 \left(\frac{8-y}{2}\right)dy = \frac{1}{2}\int_2^4 (8-y)dy$

$= \frac{1}{2}\left[8y - \frac{y^2}{2}\right]_2^4 = \frac{1}{2}\left[\left(32 - \frac{16}{2}\right) - \left(16 - \frac{4}{2}\right)\right]$

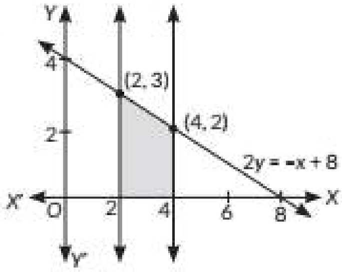
$= \frac{1}{2} \times 10 = 5$ sq.units

2. Required area $= \int_0^3 \frac{y^2}{4}dy$

$= \left[\frac{y^3}{12}\right]_0^3 = \frac{9}{4}$ sq. units.



3.

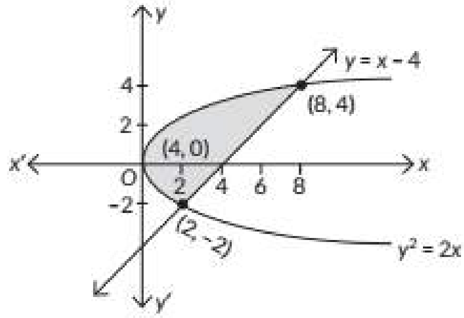


$\therefore$ Required area

$= \int_2^4 \left(\frac{-x+8}{2}\right)dx = \left[\frac{-x^2}{4} + 4x\right]_2^4 = 5$ sq. units

4. We have, $y^2 \le 2x$ ... (i)

and $y \ge x - 4$ ... (ii)



Here, the shaded area represents the required area.

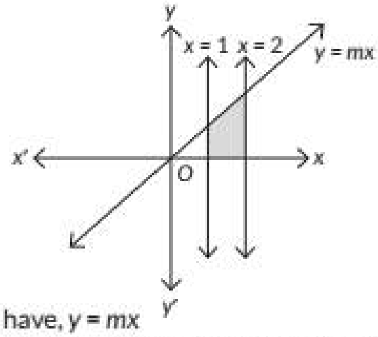
$\therefore$ Required area $= \int_{-2}^{4} (y+4)dy - \int_{-2}^{4} \frac{y^2}{2}dy$

$= \left[\frac{y^2}{2} + 4y\right]_{-2}^{4} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-2}^{4}$

$= \left[\left(\frac{16}{2} + 16\right) - \left(\frac{4}{2} - 8\right)\right] - \frac{1}{2}\left[\frac{64}{3} - \left(\frac{-8}{3}\right)\right]$

$= 30 - 12 = 18$ sq. units

5.



We have, $y = mx$

From the figure it is clear that required area is the shaded region.

Required area $= \int_1^2 mx\,dx = \left[\frac{mx^2}{2}\right]_1^2 = \frac{4m}{2} - \frac{m}{2} = \frac{3m}{2}$ sq. units.

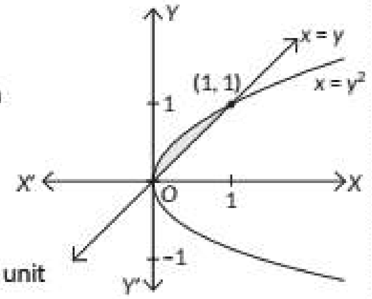
6. On solving $x = y$ and $x = y^2$, we get

$y^2 = y$

$\Rightarrow$ $y(y - 1) = 0$

$\Rightarrow$ $y = 0$ and $y = 1$.

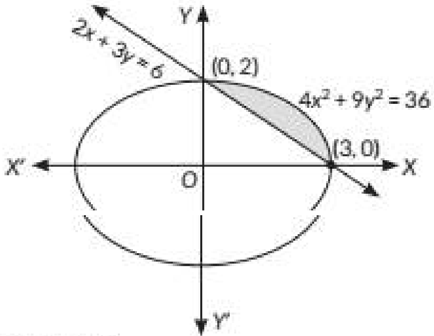
$\therefore$ Required area = area of shaded region



$= \int_0^1 (y - y^2)\,dy$

$= \left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ sq. unit

7.



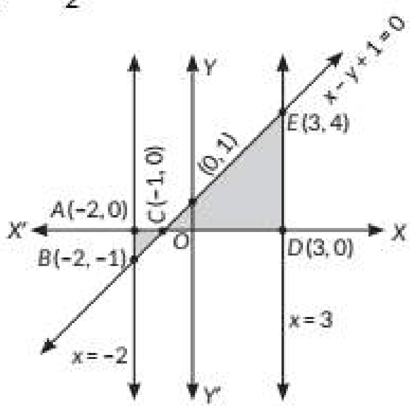
Required area

$= \int_0^3 \left(\frac{\sqrt{36-4x^2}}{3} - \left(\frac{6-2x}{3}\right)\right)dx = \frac{2}{3}\int_0^3 (\sqrt{9-x^2} - (3-x))dx$

$= \frac{2}{3}\left[\left(\frac{1}{2}x\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right) + \frac{x^2}{2} - 3x\right]_0^3$

$= \frac{3\pi}{2} - 3 = \frac{3}{2}(\pi - 2)$ sq. units

8.



Required area $= ar(\Delta ABC) + ar(\Delta CDE)$

$= \left|\int_{-2}^{-1} (x+1)dx\right| + \int_{-1}^{3} (x+1)dx = \left|\left[\frac{x^2}{2} + x\right]_{-2}^{-1}\right| + \left[\frac{x^2}{2} + x\right]_{-1}^{3}$

$= \left|\frac{1}{2} - 1 - (2-2)\right| + \left\{\frac{9}{2} + 3 - \left(\frac{1}{2} - 1\right)\right\} = \frac{1}{2} + 8 = \frac{17}{2} = 8.5$ sq. units

9. Given, area $= 2\int_0^{4a} \sqrt{4ax}\,dx = \frac{256}{3}$

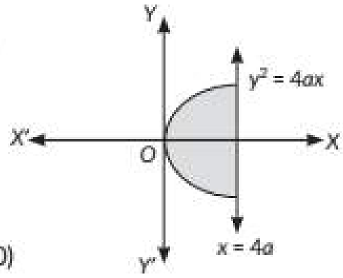
$\Rightarrow$ $4\sqrt{a}\int_0^{4a} \sqrt{x}\,dx = \frac{256}{3}$

$\Rightarrow$ $4\sqrt{a}\,\frac{2}{3}\left[(x)^{3/2}\right]_0^{4a} = \frac{256}{3}$

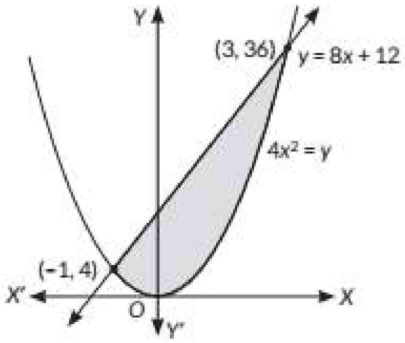
$\Rightarrow$ $\frac{8}{3}\sqrt{a}\,(4a)^{3/2} = \frac{256}{3}$

$\Rightarrow$ $a^2 = \frac{256}{3} \times \frac{3}{8 \times 2 \times 4}$

$\Rightarrow$ $a^2 = 4 \Rightarrow a = 2$ ($\because a > 0$)



10. The graph of given region is

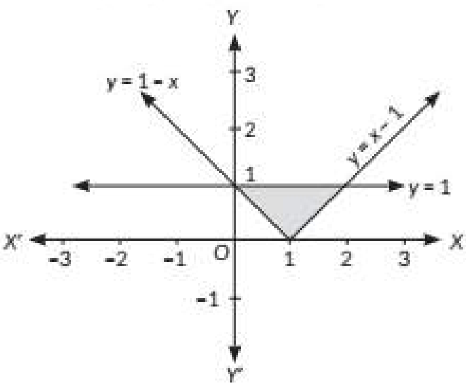


$\therefore$ Required area $= \int_{-1}^{3} (8x + 12 - 4x^2)\,dx$

$= \left[4x^2 + 12x - \frac{4}{3}x^3\right]_{-1}^{3}$

$= 36 + 36 - 36 - \left(4 - 12 + \frac{4}{3}\right) = 44 - \frac{4}{3} = \frac{128}{3}$ sq. units

11. Given curve, $y = |x - 1|$ and line $y = 1$



We have, $y = \begin{cases} x - 1, & \text{if } x - 1 \geq 0 \\ -x + 1, & \text{if } x - 1 < 0 \end{cases}$

Required area = area of shaded region

$$=\int_0^2 1\,dx-\left\{\int_0^1(1-x)dx+\int_1^2(x-1)dx\right\}$$

$$=[x]_0^2-\left(\left[x-\frac{x^2}{2}\right]_0^1+\left[\frac{x^2}{2}-x\right]_1^2\right)$$

$$=2-\left(1-\frac{1}{2}+2-2-\frac{1}{2}+1\right)$$

$$=2-\left(\frac{1}{2}+\frac{1}{2}\right)=2-1=1\text{ sq. unit}$$

12. Given, $x^2+y^2=16$ ...(i)

$y=x$ ...(ii) and $x=0$ ...(iii)

From equation (i) and (iii), we get

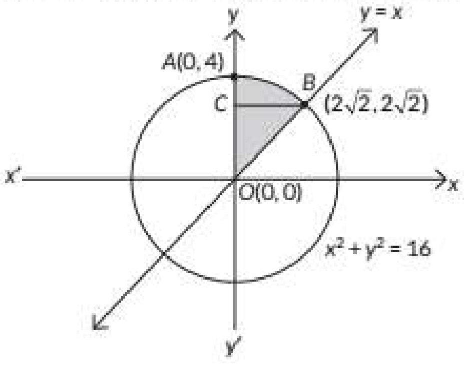
$x=0$ and $y=\pm 4$

∴ Equation (i) and (iii) intersect at (0, 4) and (0, −4).

From equation (i) and (ii), we get

$x=\pm 2\sqrt{2}$ and $y=\pm 2\sqrt{2}$

Required bounded region is shown in the figure.



Let A be the required area.

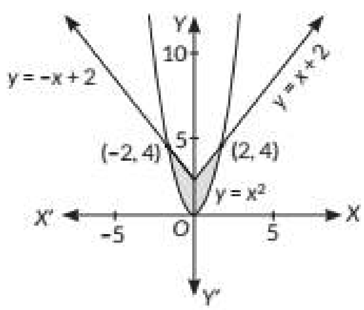
A = Area of ΔOBC + Area of region $CBAC$

$$=\int_0^{2\sqrt{2}} y\,dy+\int_{2\sqrt{2}}^{4}\sqrt{16-y^2}\,dy$$

$$=\left[\frac{y^2}{2}\right]_0^{2\sqrt{2}}+\left[\frac{y}{2}\sqrt{16-y^2}+\frac{16}{2}\sin^{-1}\left(\frac{y}{4}\right)\right]_{2\sqrt{2}}^{4}$$

$$=4+8\times\frac{\pi}{2}-4-8\times\frac{\pi}{4}=2\pi\text{ sq. units}$$

13. The graph of given region is

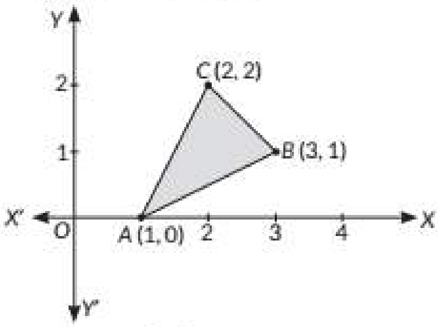


We have, $y=\begin{cases}x+2 & \text{if } x\ge 0\\ -x+2 & \text{if } x<0\end{cases}$

Required area $=2\int_0^2(x+2-x^2)dx$

$$=2\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_0^2=4+8-\frac{16}{3}=\frac{20}{3}\text{ sq. units}$$

14. The graph of given region is



Equation of AB, $y-0=\frac{1-0}{3-1}(x-1)$

$\Rightarrow\ 2y=x-1\Rightarrow\ y=\frac{x-1}{2}$

Equation of BC, $y-2=\frac{1-2}{3-2}(x-2)$

$\Rightarrow\ y=-x+4$

Equation of AC, $y-0=\frac{2-0}{2-1}(x-1)$

$\Rightarrow\ y=2x-2$

Required area

$$=\int_1^2\left[2x-2-\left(\frac{x-1}{2}\right)\right]dx+\int_2^3\left[(-x+4)-\left(\frac{x-1}{2}\right)\right]dx$$

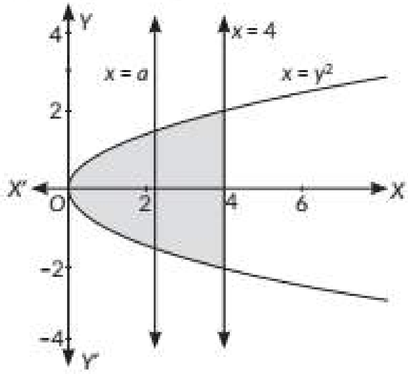
$$=\int_1^2(3x-3)dx+\int_2^3(-3x+9)dx$$

$$=3\left[\frac{x^2}{2}-x\right]_1^2+\left[\frac{-3x^2}{2}+9x\right]_2^3$$

$$=3\left(2-2-\frac{1}{2}+1\right)+\left(-\frac{27}{2}+27+6-18\right)$$

$$=\frac{3}{2}+\frac{3}{2}=3\text{ sq. units}$$

15. The graph of given region is



According to question, $2\int_0^a\sqrt{x}\,dx=2\int_a^4\sqrt{x}\,dx$

$\Rightarrow\ \frac{2}{3}\left[x^{3/2}\right]_0^a=\frac{2}{3}\left[x^{3/2}\right]_a^4$

$\Rightarrow\ a^{3/2}=8-a^{3/2}$

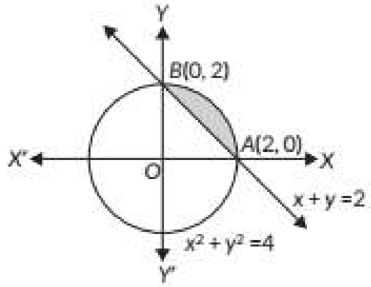
$\Rightarrow \quad 2a^{3/2} = 8$

$\Rightarrow \quad a^{3/2} = 4 \Rightarrow a = (4^2)^{1/3}$

$\Rightarrow \quad a = (16)^{1/3} = (2)^{4/3}$

16. The given curves are

$x^2 + y^2 = 4$ ...(i) and $x + y = 2$ ...(ii)



$\therefore$ Required area $= \int_0^2 \left[\sqrt{4-x^2} - (2-x)\right] dx$

$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2} - 2x + \frac{x^2}{2}\right]_0^2$

$= 0 + 2\sin^{-1}(1) - 4 + 2 - 0$

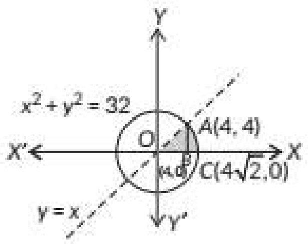
$= 2\cdot\frac{\pi}{2} - 2 = (\pi - 2)$ sq. units.

Key Points

- $\int \sqrt{a^2-x^2}\,dx = \frac{x\cdot\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

17. The given equation of the circle is $x^2 + y^2 = 32$ and the line is $y = x$.

These intersect at $A(4, 4)$ in the first quadrant. The required area is shown shaded in the figure. Points $B(4, 0)$ and $C(4\sqrt{2}, 0)$.



$\therefore$ Required area = Area $BACB$ + Area $OABO$

$= \int_4^{4\sqrt{2}} y_1\,dx + \int_0^4 y_2\,dx = \int_4^{4\sqrt{2}} \sqrt{32-x^2}\,dx + \int_0^4 x\,dx$

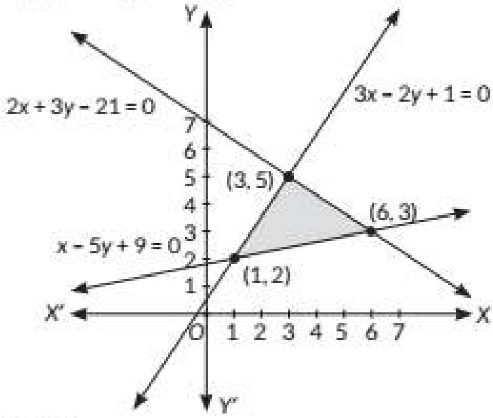
$= \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx + \int_0^4 x\,dx$

$= \left[\frac{x\sqrt{32-x^2}}{2} + \frac{32}{2}\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}} + \left[\frac{x^2}{2}\right]_0^4$

$= \frac{4\sqrt{2}\times 0}{2} + 16\sin^{-1}1 - \left(\frac{4\times 4}{2} + 16\sin^{-1}\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(4^2 - 0)$

$= 16\cdot\frac{\pi}{2} - \left(8 + 16\cdot\frac{\pi}{4}\right) + 8 = 16\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 4\pi$ sq.units

18. The graph of given region is



Required area

$= \int_1^3 \left(\frac{3x+1}{2} - \frac{x+9}{5}\right)dx + \int_3^6 \left(\frac{21-2x}{3} - \frac{x+9}{5}\right)dx$

$= \left[\frac{3x^2}{4} + \frac{x}{2} - \frac{x^2}{10} - \frac{9}{5}x\right]_1^3 + \left[7x - \frac{x^2}{3} - \frac{x^2}{10} - \frac{9}{5}x\right]_3^6$

$= \frac{13}{5} + \frac{39}{10} = \frac{65}{10} = \frac{13}{2}$ sq.units

19. We have curves, $y = \frac{1}{\sqrt{3}}x$ ...(i)

and $x^2 + y^2 = 16$ ...(ii)

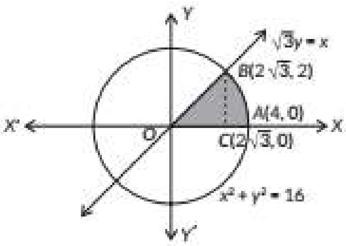
Curves (i) and (ii) intersect at $(2\sqrt{3}, 2)$ and $(-2\sqrt{3}, -2)$.

$\therefore$ Required area = Area of region $OBAO$

= area of ΔOBC + area of region $BCAB$

$= \int_0^{2\sqrt{3}} \frac{x}{\sqrt{3}}dx + \int_{2\sqrt{3}}^4 \sqrt{16-x^2}\,dx$

$= \left[\frac{x^2}{2\sqrt{3}}\right]_0^{2\sqrt{3}} + \left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{3}}^4$



$= 2\sqrt{3} + 8\left(\frac{\pi}{2}\right) - 2\sqrt{3} - \frac{8\pi}{3} = \frac{12\pi - 8\pi}{3} = \frac{4\pi}{3}$ sq.units

Commonly Made Mistake

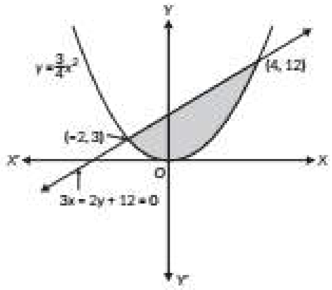
- Remember difference between the formula for $\int\sqrt{a^2-x^2}\,dx$ and $\int\sqrt{x^2-a^2}\,dx$.

20. Given equations are $y = \frac{3x^2}{4}$ ...(i)

and $3x - 2y + 12 = 0 \Rightarrow y = \frac{3x+12}{2}$ ...(ii)

Solving equations (i) and (ii), we get

$\frac{3x^2}{4} = \frac{3x+12}{2}$



$\Rightarrow\ x^2-2x-8=0 \Rightarrow (x+2)(x-4)=0 \Rightarrow x=-2, 4$

When $x=-2 \Rightarrow y=3$

When $x=4 \Rightarrow y=12$

$\therefore$ Required area $=\displaystyle\int_{-2}^{4}\left(\frac{3x+12}{2}-\frac{3}{4}x^2\right)dx$

$=\left[\frac{3}{4}x^2+6x-\frac{x^3}{4}\right]_{-2}^{4}$

$=\left[\frac{3\times16}{4}+6\times4-\frac{64}{4}\right]-\left[\frac{3}{4}\times4-6\times2+\frac{8}{4}\right]=27$ sq. units.

21. We have curves $x-y+2=0$ and $x=\sqrt{y}$.

$\Rightarrow\ y=x^2$, which is a parabola with vertex at origin.

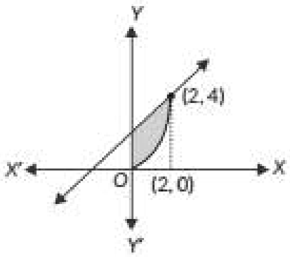
From the given equations, we get

$x-x^2+2=0 \Rightarrow (x-2)(x+1)=0$

$\Rightarrow x=2$ or $x=-1 \Rightarrow x=2$ $\quad[\because x\neq-1, x$ is positive$]$

When $x=2, y=4$

So, the point of intersection is (2, 4).



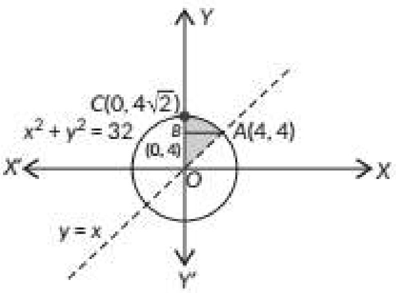
$\therefore$ Required area $=\displaystyle\int_0^2(x+2)dx-\int_0^2 x^2dx=\int_0^2(x+2-x^2)dx$

$=\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_0^2$

$=2+4-\frac{8}{3}=\frac{10}{3}$ sq.units

22. The given equation of the circle is $x^2+y^2=32$ and the line is $y=x$.

These intersect at $A(4, 4)$ in the first quadrant. The required area is shown shaded in the figure. Points $B(0, 4)$ and $C(0, 4\sqrt{2})$.



$\therefore$ Required area = Area $BACB$ + Area $OABO$

$=\displaystyle\int_4^{4\sqrt{2}} x_1dy+\int_0^4 x_2dy$

$=\displaystyle\int_4^{4\sqrt{2}}\sqrt{32-y^2}\,dy+\int_0^4 y\,dy$

$=\displaystyle\int_4^{4\sqrt{2}}\sqrt{(4\sqrt{2})^2-y^2}\,dy+\int_0^4 y\,dy$

$=\left[\frac{y\sqrt{32-y^2}}{2}+\frac{32}{2}\sin^{-1}\left(\frac{y}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}+\left[\frac{y^2}{2}\right]_0^4$

$=\frac{4\sqrt{2}\times0}{2}+16\sin^{-1}1-\left(\frac{4\times4}{2}+16\sin^{-1}\frac{1}{\sqrt{2}}\right)+\frac{1}{2}(4^2-0)$

$=16\cdot\frac{\pi}{2}-\left(8+16\cdot\frac{\pi}{4}\right)+8=16\left(\frac{\pi}{2}-\frac{\pi}{4}\right)=4\pi$ sq.units

Concept Applied

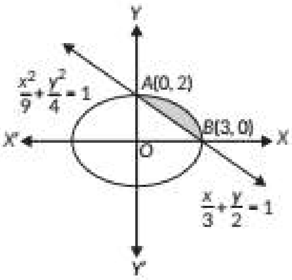
- Area of the region bounded by the curve $x=g(y)$, y-axis and the lines $y=a$ and $y=b$ $(b>a)$ is,

  Area $=\displaystyle\int_a^b x\,dy=\int_a^b g(y)\,dy$

23. We have $\frac{x^2}{9}+\frac{y^2}{4}=1$ ...(i) and $\frac{x}{3}+\frac{y}{2}=1$ ...(ii)

Curve (i) is an ellipse of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

That means its major axis is along x – axis. Also this ellipse is symmetrical about the x – axis.



Required area $=\frac{2}{3}\displaystyle\int_0^3\sqrt{(3)^2-x^2}\,dx-\frac{2}{3}\int_0^3(3-x)dx$

$=\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3-\frac{2}{3}\left[\frac{(3-x)^2}{-2}\right]_0^3$

$=\frac{2}{3}\left[\left(0+\frac{9}{2}\sin^{-1}(1)\right)-\left(0+\frac{9}{2}\sin^{-1}(0)\right)\right]+\frac{1}{3}[0^2-9]$

$=\frac{3\pi}{2}-3$ sq. units.

24. Here, $y=|x+1|+1$

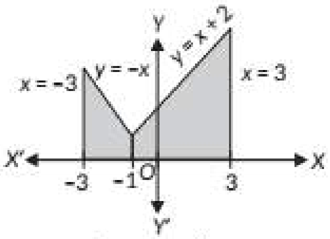
$$y=\begin{cases}x+2 & \text{if } x\geq-1\\ -x & \text{if } x<-1\end{cases}$$

We know draw the lines : $y = 0, x = 3, x = -3$ and

$y = x + 2$ if $x \geq -1$ ...(i)

$y = -x$ if $x < -1$ ...(ii)

Lines (i) and (ii) intersect at $(-1, 1)$



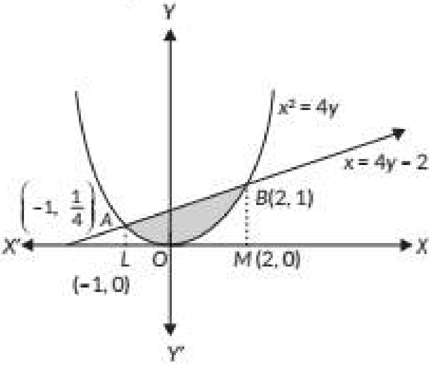
$\therefore$ Required area $= \int_{-3}^{-1} (-x)dx + \int_{-1}^{3} (x+2)dx$

$$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3} = -\frac{1}{2}(1-9) + \frac{1}{2}(9-1) + 2(3+1)$$

$= 4 + 4 + 8 = 16$ sq. units.

25. The given curve is $x^2 = 4y$ ...(i)

The given line is $x = 4y - 2$ ...(ii)



Putting $4y = (x + 2)$ from (ii) in (i), we get $(x + 2) = x^2$

$\Rightarrow \quad x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = 2, -1$

Putting $x = 2$ in (i), we get $y = 1$

Putting $x = -1$ in (i), we get $y = \frac{1}{4}$

Thus the points of intersection of the given curve and line are $A\left(-1, \frac{1}{4}\right)$ and $B(2, 1)$.

$\therefore$ Required area

$$= \int_{-1}^{2} \left(\frac{x+2}{4}\right) dx - \int_{-1}^{2} \frac{x^2}{4} dx = \int_{-1}^{2} \left(\frac{x}{4} + \frac{1}{2} - \frac{x^2}{4}\right) dx$$

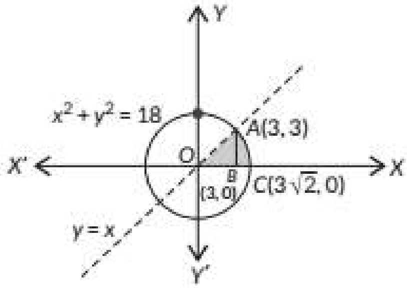
$$= \frac{1}{4}\left[\frac{x^2}{2}\right]_{-1}^{2} + \frac{1}{2}[x]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{8}[4-1] + \frac{1}{2}[2+1] - \frac{1}{12}[8+1]$$

$$= \frac{3}{8} + \frac{3}{2} - \frac{3}{4} = \frac{3}{2}\left[\frac{1}{4} + 1 - \frac{1}{2}\right] = \frac{3}{2}\left[\frac{3}{4}\right] = \frac{9}{8} \text{ sq. units}$$

26. The given equation of the circle is $x^2 + y^2 = 18$ and the line is $y = x$.

These intersect at $A(3, 3)$ in the first quadrant. The required area is shown shaded in the figure. Points $B(3, 0)$ and $C(3\sqrt{2}, 0)$.



$\therefore$ Required area = Area $BACB$ + Area $OABO$

$$= \int_{3}^{3\sqrt{2}} y_1 dx + \int_0^3 y_2 dx = \int_{3}^{3\sqrt{2}} \sqrt{18 - x^2}\, dx + \int_0^3 x\, dx$$

$$= \int_{3}^{3\sqrt{2}} \sqrt{(3\sqrt{2})^2 - x^2}\, dx + \int_0^3 x\, dx$$

$$= \left[\frac{x\sqrt{18-x^2}}{2} + \frac{18}{2}\sin^{-1}\left(\frac{x}{3\sqrt{2}}\right)\right]_3^{3\sqrt{2}} + \left[\frac{x^2}{2}\right]_0^3$$

$$= \frac{3\sqrt{2} \times 0}{2} + 9\sin^{-1} 1 - \left(\frac{3 \times 3}{2} + 9\sin^{-1}\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(9 - 0)$$

$$= 9 \cdot \frac{\pi}{2} - \left(\frac{9}{2} + 9 \cdot \frac{\pi}{4}\right) + \frac{9}{2} = 9\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{9\pi}{4} \text{ sq.units}$$

Answer Tips

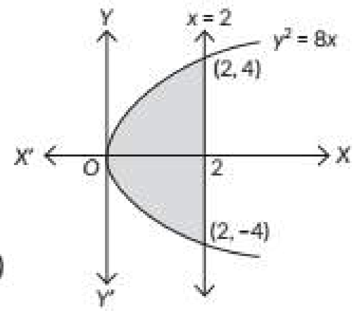
- Area of the region in the first quadrant enclosed by x-axis, the line $y = x$ and circle $x^2 + y^2 = a^2$ is $\frac{\pi a^2}{8}$.

CBSE Sample Questions

1. Required area, $A = \int_{-1}^{1} x^2 \, dx$

$$\Rightarrow A = 2\int_0^1 x^2 dx = \frac{2}{3}\left[x^3\right]_0^1 = \frac{2}{3} \text{ sq.units} \qquad (1)$$

2. Required area



$$= 2\int_0^2 \sqrt{8x}\, dx \qquad (1)$$

$$= 2 \times 2\sqrt{2} \int_0^2 x^{1/2} dx$$

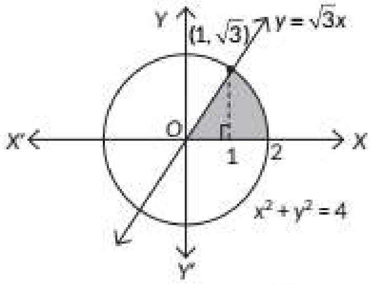
$$= 4\sqrt{2}\left[\frac{2}{3}x^{3/2}\right]_0^2 \qquad (1/2)$$

$$= \frac{8}{3}\sqrt{2}\left[2^{3/2} - 0\right] = \frac{8\sqrt{2}}{3} \times 2\sqrt{2} = \frac{32}{3} \text{ sq.units} \qquad (1/2)$$

3. Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get

$x^2 + 3x^2 = 4$

$\Rightarrow \quad x^2 = 1 \Rightarrow x = 1$ (1/2)



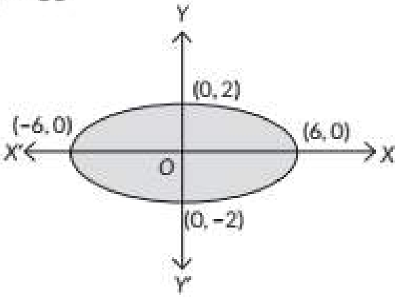
(1/2)

$\therefore$ Required area $= \sqrt{3}\int_0^1 x\,dx + \int_1^2 \sqrt{2^2 - x^2}\,dx$ (1/2)

$= \frac{\sqrt{3}}{2}[x^2]_0^1 + \left[\frac{x}{2}\sqrt{2^2-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$ (1)

$= \frac{\sqrt{3}}{2} + \left[2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{6}\right] = \frac{2\pi}{3}$ sq. units (1/2)

4. Given equation of ellipse is
$x^2 + 9y^2 = 36$



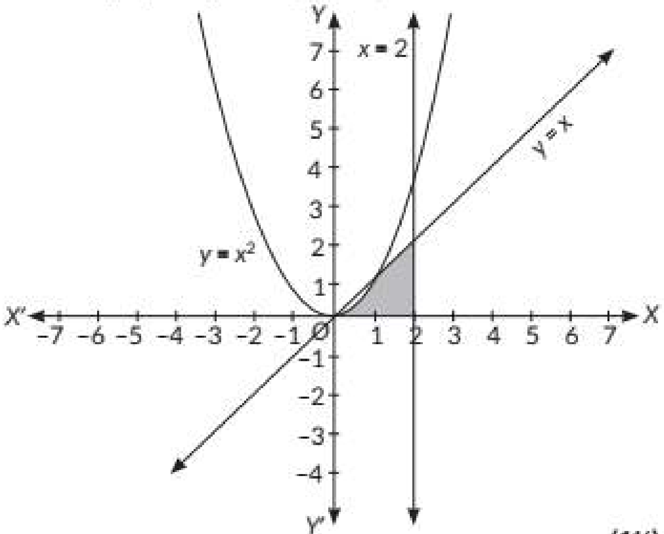
$\therefore$ Required area $= 4\int_0^6 \sqrt{\frac{36-x^2}{9}}\,dx$ (1/2)

$= \frac{4}{3}\int_0^6 \sqrt{6^2-x^2}\,dx$ (1/2)

$= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2-x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$ (1)

$= \frac{4}{3}\left[18\times\frac{\pi}{2} - 0\right] = 12\pi$ sq. units (1)

5. The graph of given region is;



(1½)

The points of intersection of the parabola $y = x^2$ and the line $y = x$ are

$x^2 = x$

$\Rightarrow$ $x(x - 1) = 0$

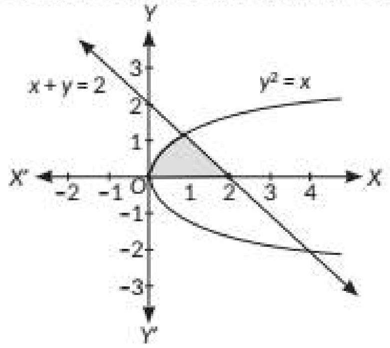
$\Rightarrow$ $x = 0, 1$ (1)

So, point of intersection is (0, 0) and (1, 1).

Required area $= \int_0^1 x^2\,dx + \int_1^2 x\,dx$ (1)

$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$ sq. units (1½)

6. By solving $x + y = 2$ and $y^2 = x$ simultaneously, we get the points of intersection as (1, 1) and (4, –2). (1)



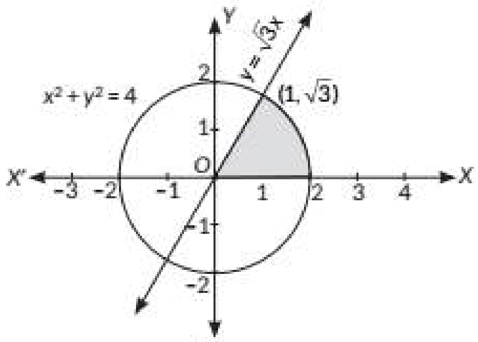
(1)

$\therefore$ Required area = The shaded area

$= \int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)\,dx$ (1)

$= \frac{2}{3}[x^{3/2}]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2 = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$ square units. (1)

7. By solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get the points of intersection as $(1, \sqrt{3})$ and $(-1, -\sqrt{3})$. (1)



(1)

$\therefore$ Required area = The shaded area

$= \int_0^1 \sqrt{3}x\,dx + \int_1^2 \sqrt{4-x^2}\,dx$

$= \frac{\sqrt{3}}{2}[x^2]_0^1 + \frac{1}{2}\left[x\sqrt{4-x^2} + 4\sin^{-1}\frac{x}{2}\right]_1^2$ (1)

$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left[2\pi - \sqrt{3} - 2\times\frac{\pi}{3}\right]$

$= \frac{2\pi}{3}$ square units. (1)